Review of Multi-Agent Algorithms for Collective Behavior: a Structural Taxonomy

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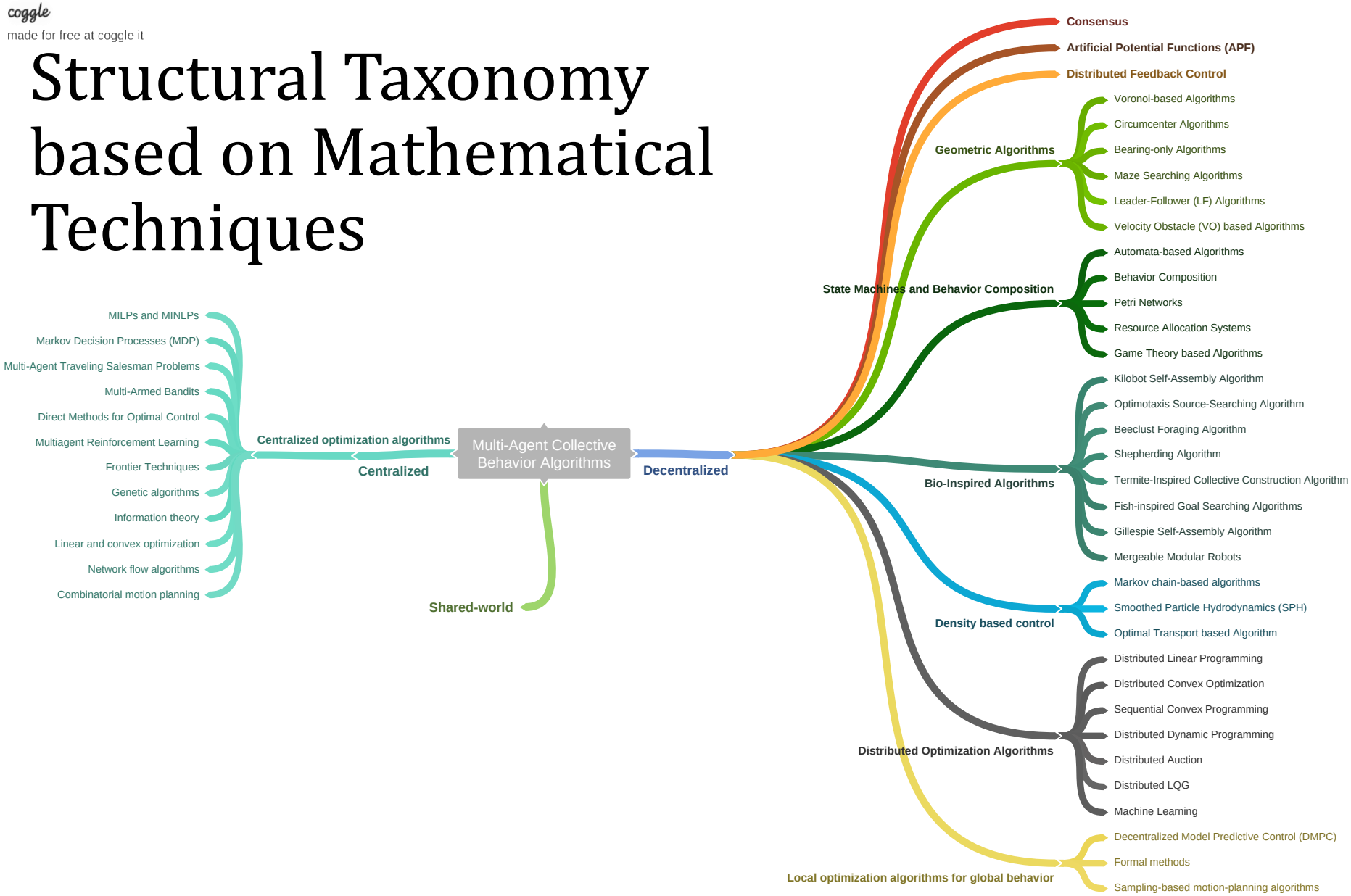
Structural Taxonomy based on Mathematical Techniques



F. Rossi, S. Bandyopadhyay, M. Wolf, M. Pavone, "Review of Multi-Agent Algorithms for Collective Behavior: a Structural Taxonomy", https://arxiv.org/abs/1803.05464

Motivation

- Select algorithms for a given task or application
- Same algorithm can be used for a variety of tasks
- Identify areas for future research

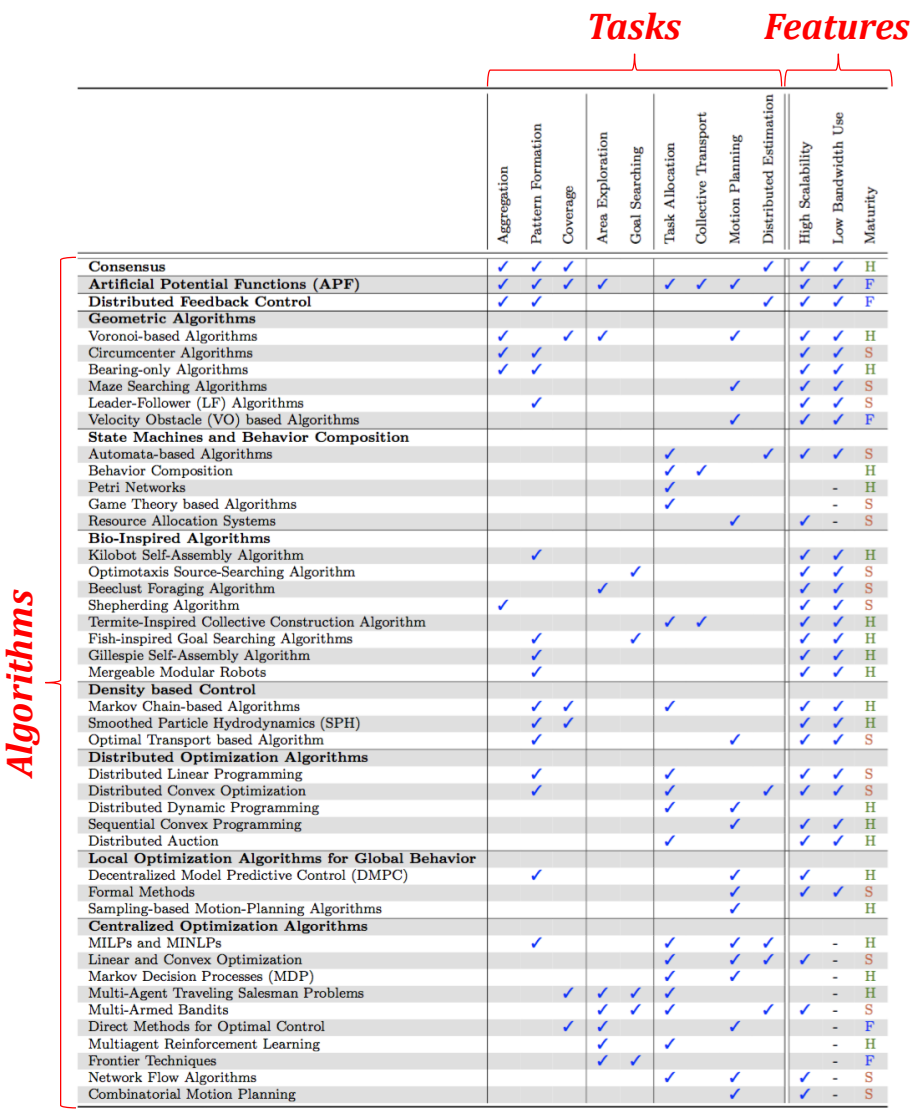
Tasks *Features*

Algorithms

	Aggregation	Pattern Formation	Coverage	Area Exploration	Goal Searching	Task Allocation	Collective Transport	Motion Planning	Distributed Estimation	High Scalability	Low Bandwidth Use	Maturity
Consensus	✓	✓	✓						✓	✓	✓	H
Artificial Potential Functions (APF)	✓	✓	✓	✓		✓	✓	✓		✓	✓	F
Distributed Feedback Control	✓	✓							✓	✓	✓	F
Geometric Algorithms												
Voronoi-based Algorithms	✓		✓	✓				✓		✓	✓	H
Circumcenter Algorithms	✓	✓								✓	✓	S
Bearing-only Algorithms	✓	✓								✓	✓	H
Maze Searching Algorithms								✓		✓	✓	S
Leader-Follower (LF) Algorithms		✓								✓	✓	S
Velocity Obstacle (VO) based Algorithms								✓		✓	✓	F
State Machines and Behavior Composition												
Automata-based Algorithms						✓			✓	✓	✓	S
Behavior Composition						✓	✓					H
Petri Networks						✓					-	H
Game Theory based Algorithms						✓					-	S
Resource Allocation Systems								✓		✓	-	S
Bio-Inspired Algorithms												
Kilobot Self-Assembly Algorithm		✓								✓	✓	H
Optimotaxis Source-Searching Algorithm					✓					✓	✓	S
Beeclust Foraging Algorithm				✓						✓	✓	S
Shepherding Algorithm	✓									✓	✓	S
Termite-Inspired Collective Construction Algorithm						✓	✓			✓	✓	H
Fish-inspired Goal Searching Algorithms		✓			✓					✓	✓	H
Gillespie Self-Assembly Algorithm		✓								✓	✓	H
Mergeable Modular Robots		✓								✓	✓	H
Density based Control												
Markov Chain-based Algorithms		✓	✓			✓				✓	✓	H
Smoothed Particle Hydrodynamics (SPH)		✓	✓							✓	✓	H
Optimal Transport based Algorithm		✓						✓		✓	✓	S
Distributed Optimization Algorithms												
Distributed Linear Programming		✓				✓				✓	✓	S
Distributed Convex Optimization		✓				✓			✓	✓	✓	S
Distributed Dynamic Programming						✓		✓				H
Sequential Convex Programming								✓		✓	✓	H
Distributed Auction						✓				✓	✓	H
Local Optimization Algorithms for Global Behavior												
Decentralized Model Predictive Control (DMPC)		✓						✓		✓		H
Formal Methods								✓		✓	✓	S
Sampling-based Motion-Planning Algorithms								✓				H
Centralized Optimization Algorithms												
MILPs and MINLPs		✓				✓		✓	✓		-	H
Linear and Convex Optimization						✓		✓	✓	✓	-	S
Markov Decision Processes (MDP)						✓		✓			-	H
Multi-Agent Traveling Salesman Problems			✓	✓	✓	✓					-	H
Multi-Armed Bandits				✓	✓	✓			✓	✓	-	S
Direct Methods for Optimal Control			✓	✓				✓			-	F
Multiagent Reinforcement Learning				✓		✓					-	H
Frontier Techniques				✓	✓						-	F
Network Flow Algorithms						✓		✓		✓	-	S
Combinatorial Motion Planning								✓		✓	-	S

Tasks in multi-agent systems

- Spatially-organizing behaviors
  - Aggregation
  - Pattern Formation
  - Coverage
- Collective explorations
  - Area Exploration
  - Goal Searching
- Cooperative decision making
  - Task Allocation
  - Collective Transport
  - Motion Planning
  - Distributed Estimation



[1] Brambilla M, Ferrante E, Birattari M and Dorigo M, "Swarm robotics: a review from the swarm engineering perspective". *Swarm Intelligence* 7(1): 1–41, 2013.

Features of Algorithms

- Scalability
- Bandwidth use
- Maturity

Algorithms	Tasks: Aggregation	Pattern Formation	Coverage	Area Exploration	Goal Searching	Task Allocation	Collective Transport	Motion Planning	Distributed Estimation	Features: High Scalability	Low Bandwidth Use	Maturity
Consensus	✓	✓	✓						✓	✓	✓	H
Artificial Potential Functions (APF)	✓	✓	✓	✓		✓	✓	✓		✓	✓	F
Distributed Feedback Control	✓	✓							✓	✓	✓	F
Geometric Algorithms												
Voronoi-based Algorithms	✓		✓	✓				✓		✓	✓	H
Circumcenter Algorithms	✓	✓								✓	✓	S
Bearing-only Algorithms	✓	✓								✓	✓	H
Maze Searching Algorithms								✓		✓	✓	S
Leader-Follower (LF) Algorithms		✓								✓	✓	S
Velocity Obstacle (VO) based Algorithms								✓		✓	✓	F
State Machines and Behavior Composition												
Automata-based Algorithms						✓			✓	✓	✓	S
Behavior Composition						✓	✓					H
Petri Networks						✓					-	H
Game Theory based Algorithms						✓					-	S
Resource Allocation Systems								✓		✓	-	S
Bio-Inspired Algorithms												
Kilobot Self-Assembly Algorithm		✓								✓	✓	H
Optimotaxis Source-Searching Algorithm					✓					✓	✓	S
Beeclust Foraging Algorithm				✓						✓	✓	S
Shepherding Algorithm	✓									✓	✓	S
Termite-Inspired Collective Construction Algorithm						✓	✓			✓	✓	H
Fish-inspired Goal Searching Algorithms		✓			✓					✓	✓	H
Gillespie Self-Assembly Algorithm		✓								✓	✓	H
Mergeable Modular Robots		✓								✓	✓	H
Density based Control												
Markov Chain-based Algorithms		✓	✓			✓				✓	✓	H
Smoothed Particle Hydrodynamics (SPH)		✓	✓							✓	✓	H
Optimal Transport based Algorithm		✓						✓		✓	✓	S
Distributed Optimization Algorithms												
Distributed Linear Programming		✓				✓				✓	✓	S
Distributed Convex Optimization		✓				✓			✓	✓	✓	S
Distributed Dynamic Programming						✓		✓				H
Sequential Convex Programming								✓		✓	✓	H
Distributed Auction						✓				✓	✓	H
Local Optimization Algorithms for Global Behavior												
Decentralized Model Predictive Control (DMPC)		✓						✓		✓		H
Formal Methods								✓		✓	✓	S
Sampling-based Motion-Planning Algorithms								✓				H
Centralized Optimization Algorithms												
MILPs and MINLPs		✓				✓		✓	✓		-	H
Linear and Convex Optimization						✓		✓	✓	✓	-	S
Markov Decision Processes (MDP)						✓		✓			-	H
Multi-Agent Traveling Salesman Problems			✓	✓	✓	✓					-	H
Multi-Armed Bandits				✓	✓	✓			✓	✓	-	S
Direct Methods for Optimal Control			✓	✓				✓			-	F
Multiagent Reinforcement Learning				✓		✓					-	H
Frontier Techniques				✓	✓						-	F
Network Flow Algorithms						✓		✓		✓	-	S
Combinatorial Motion Planning								✓		✓	-	S

Example: Consensus Algorithm

- **Mathematical description:** $x_{k+1}^i = \sum_{j \in \mathcal{J}_k^i} A_k[i,j] x_k^j$

[1] Tsitsiklis JN, Bertsekas DP and Athans M, "Distributed asynchronous deterministic and stochastic gradient optimization algorithms", *IEEE TAC*, 1986.
[2] Jadbabaie A, Lin J and Morse AS, "Coordination of groups of mobile autonomous agents using nearest neighbor rules", *IEEE TAC*, 2003.
[3] Olfati-Saber R and Murray R, "Consensus problems in networks of agents with switching topology and time-delays", *IEEE TAC*, 2004.
[4] Ren W and Beard RW, "Consensus seeking in multiagent systems under dynamically changing interaction topologies", *IEEE TAC*, 2005.

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- **Mathematical description:** $x_{k+1}^i = \sum_{j \in \mathcal{J}_k^i} A_k[i,j] x_k^j$
- **Mathematical guarantees:** Connected undirected networks with first-order linear dynamics [1], Time-varying network topology [2], Arbitrary network topology [3], Random networks [4].

  Communication delays [5], Quantized information [6].

  Second-order linear dynamics [7], nonlinear Lagrangian dynamics [8]

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[2] Ren W and Beard RW, "Consensus seeking in multiagent systems under dynamically changing interaction topologies", *IEEE TAC*, 2005.
[3] Qin J, Yu C and Gao H, "Collective behavior for group of generic linear agents interacting under arbitrary network topology", *IEEE Transactions on Control of Network Systems*, 2015.
[4] Tahbaz-Salehi A and Jadbabaie A, "A necessary and sufficient condition for consensus over random networks", *IEEE TAC*, 2008.
[5] Spanos DP, Olfati-Saber R and Murray RM, "Dynamic consensus on mobile networks", In: Proc. IFAC, 2005.
[6] Nedic A, Olshevsky A, Ozdaglar A and Tsitsiklis JN, "On distributed averaging algorithms and quantization effects", *IEEE TAC*, 2009.
[7] Yu W, Chen G and Cao M, "Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems", *Automatica*, 2010.
[8] Chung SJ and Slotine JJE, "Cooperative robot control and concurrent synchronization of Lagrangian systems", *IEEE TRO*, 2009.

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	Aggregation	Pattern Formation	Coverage	Area Exploration	Goal Searching	Task Allocation	Collective Transport	Motion Planning	Distributed Estimation	High Scalability	Low Bandwidth Use	Maturity
Consensus	✓	✓	✓						✓	✓	✓	H

Classification of Algorithms

	Aggregation	Pattern Formation	Coverage	Area Exploration	Goal Searching	Task Allocation	Collective Transport	Motion Planning	Distributed Estimation	High Scalability	Low Bandwidth Use	Maturity
Consensus	✓	✓	✓						✓	✓	✓	H
Artificial Potential Functions (APF)	✓	✓	✓	✓		✓	✓	✓		✓	✓	F
Distributed Feedback Control	✓	✓							✓	✓	✓	F
Geometric Algorithms												
Voronoi-based Algorithms	✓		✓	✓				✓		✓	✓	H
Circumcenter Algorithms	✓	✓								✓	✓	S
Bearing-only Algorithms	✓	✓								✓	✓	H
Maze Searching Algorithms								✓		✓	✓	S
Leader-Follower (LF) Algorithms		✓								✓	✓	S
Velocity Obstacle (VO) based Algorithms								✓		✓	✓	F
State Machines and Behavior Composition												
Automata-based Algorithms						✓			✓	✓	✓	S
Behavior Composition						✓	✓					H
Petri Networks						✓					-	H
Game Theory based Algorithms						✓					-	S
Resource Allocation Systems								✓		✓	-	S

Classification of Algorithms

	Aggregation	Pattern Formation	Coverage	Area Exploration	Goal Searching	Task Allocation	Collective Transport	Motion Planning	Distributed Estimation	High Scalability	Low Bandwidth Use	Maturity
Bio-Inspired Algorithms												
Kilobot Self-Assembly Algorithm		✓								✓	✓	H
Optimotaxis Source-Searching Algorithm					✓					✓	✓	S
Beeclust Foraging Algorithm				✓						✓	✓	S
Shepherding Algorithm	✓									✓	✓	S
Termite-Inspired Collective Construction Algorithm						✓	✓			✓	✓	H
Fish-inspired Goal Searching Algorithms		✓			✓					✓	✓	H
Gillespie Self-Assembly Algorithm		✓								✓	✓	H
Mergeable Modular Robots		✓								✓	✓	H
Density based Control												
Markov Chain-based Algorithms		✓	✓			✓				✓	✓	H
Smoothed Particle Hydrodynamics (SPH)		✓	✓							✓	✓	H
Optimal Transport based Algorithm		✓						✓		✓	✓	S
Distributed Optimization Algorithms												
Distributed Linear Programming		✓				✓				✓	✓	S
Distributed Convex Optimization		✓				✓			✓	✓	✓	S
Distributed Dynamic Programming						✓		✓				H
Sequential Convex Programming								✓		✓	✓	H
Distributed Auction						✓				✓	✓	H

Classification of Algorithms

	Aggregation	Pattern Formation	Coverage	Area Exploration	Goal Searching	Task Allocation	Collective Transport	Motion Planning	Distributed Estimation	High Scalability	Low Bandwidth Use	Maturity
Local Optimization Algorithms for Global Behavior												
Decentralized Model Predictive Control (DMPC)		✓						✓		✓		H
Formal Methods								✓		✓	✓	S
Sampling-based Motion-Planning Algorithms								✓				H
Centralized Optimization Algorithms												
MILPs and MINLPs		✓				✓		✓	✓		-	H
Linear and Convex Optimization						✓		✓	✓	✓	-	S
Markov Decision Processes (MDP)						✓		✓			-	H
Multi-Agent Traveling Salesman Problems			✓	✓	✓	✓					-	H
Multi-Armed Bandits				✓	✓	✓			✓	✓	-	S
Direct Methods for Optimal Control			✓	✓				✓			-	F
Multiagent Reinforcement Learning				✓		✓					-	H
Frontier Techniques				✓	✓						-	F
Network Flow Algorithms						✓		✓		✓	-	S
Combinatorial Motion Planning								✓		✓	-	S

Example: Markov Chain-based Algorithms

	Aggregation	Pattern Formation	Coverage	Area Exploration	Goal Searching	Task Allocation	Collective Transport	Motion Planning	Distributed Estimation	High Scalability	Low Bandwidth Use	Maturity
Consensus	✓	✓	✓						✓	✓	✓	H
Artificial Potential Functions (APF)	✓	✓	✓	✓		✓	✓	✓		✓	✓	F
Distributed Feedback Control	✓	✓							✓	✓	✓	F
Geometric Algorithms												
Voronoi-based Algorithms	✓		✓	✓				✓		✓	✓	H
Circumcenter Algorithms	✓	✓								✓	✓	S
Bearing-only Algorithms	✓	✓								✓	✓	H
Maze Searching Algorithms								✓		✓	✓	S
Leader-Follower (LF) Algorithms		✓								✓	✓	S
Velocity Obstacle (VO) based Algorithms								✓		✓	✓	F
State Machines and Behavior Composition												
Automata-based Algorithms						✓			✓	✓	✓	S
Behavior Composition						✓	✓					H
Petri Networks						✓					-	H
Game Theory based Algorithms						✓					-	S
Resource Allocation Systems								✓		✓	-	S
Bio-Inspired Algorithms												
Kilobot Self-Assembly Algorithm		✓								✓	✓	H
Optimotaxis Source-Searching Algorithm					✓					✓	✓	S
Beeclust Foraging Algorithm				✓						✓	✓	S
Shepherding Algorithm	✓									✓	✓	S
Termite-Inspired Collective Construction Algorithm						✓	✓			✓	✓	H
Fish-inspired Goal Searching Algorithms		✓			✓					✓	✓	H
Gillespie Self-Assembly Algorithm		✓								✓	✓	H
Mergeable Modular Robots		✓								✓	✓	H
Density based Control												
Markov Chain-based Algorithms		✓	✓			✓				✓	✓	H
Smoothed Particle Hydrodynamics (SPH)		✓	✓							✓	✓	H
Optimal Transport based Algorithm		✓						✓		✓	✓	S
Distributed Optimization Algorithms												
Distributed Linear Programming		✓				✓				✓	✓	S
Distributed Convex Optimization		✓				✓			✓	✓	✓	S
Distributed Dynamic Programming						✓		✓				H
Sequential Convex Programming								✓		✓	✓	H
Distributed Auction						✓				✓	✓	H
Local Optimization Algorithms for Global Behavior												
Decentralized Model Predictive Control (DMPC)		✓						✓		✓		H
Formal Methods								✓		✓	✓	S
Sampling-based Motion-Planning Algorithms								✓				H
Centralized Optimization Algorithms												
MILPs and MINLPs		✓				✓		✓	✓		-	H
Linear and Convex Optimization						✓		✓	✓	✓	-	S
Markov Decision Processes (MDP)						✓		✓			-	H
Multi-Agent Traveling Salesman Problems			✓	✓	✓	✓					-	H
Multi-Armed Bandits				✓	✓	✓			✓	✓	-	S
Direct Methods for Optimal Control			✓	✓				✓			-	F
Multiagent Reinforcement Learning				✓		✓					-	H
Frontier Techniques				✓	✓						-	F
Network Flow Algorithms						✓		✓		✓	-	S
Combinatorial Motion Planning								✓		✓	-	S

Example: Markov Chain-based Algorithms

	Aggregation	Pattern Formation	Coverage	Area Exploration	Goal Searching	Task Allocation	Collective Transport	Motion Planning	Distributed Estimation	High Scalability	Low Bandwidth Use	Maturity
Density based Control												
Markov Chain-based Algorithms		✓	✓			✓				✓	✓	H
Smoothed Particle Hydrodynamics (SPH)		✓	✓							✓	✓	H
Optimal Transport based Algorithm		✓						✓		✓	✓	S

Example: Distributed Estimation Task

	Aggregation	Pattern Formation	Coverage	Area Exploration	Goal Searching	Task Allocation	Collective Transport	Motion Planning	Distributed Estimation	High Scalability	Low Bandwidth Use	Maturity
Consensus	✓	✓	✓						✓	✓	✓	H
Artificial Potential Functions (APF)	✓	✓	✓	✓		✓	✓	✓		✓	✓	F
Distributed Feedback Control	✓	✓							✓	✓	✓	F
Geometric Algorithms												
Voronoi-based Algorithms	✓		✓	✓				✓		✓	✓	H
Circumcenter Algorithms	✓	✓								✓	✓	S
Bearing-only Algorithms	✓	✓								✓	✓	H
Maze Searching Algorithms								✓		✓	✓	S
Leader-Follower (LF) Algorithms		✓								✓	✓	S
Velocity Obstacle (VO) based Algorithms								✓		✓	✓	F
State Machines and Behavior Composition												
Automata-based Algorithms						✓			✓	✓	✓	S
Behavior Composition						✓	✓					H
Petri Networks						✓					-	H
Game Theory based Algorithms						✓					-	S
Resource Allocation Systems								✓		✓	-	S
Bio-Inspired Algorithms												
Kilobot Self-Assembly Algorithm		✓								✓	✓	H
Optimotaxis Source-Searching Algorithm					✓					✓	✓	S
Beeclust Foraging Algorithm				✓						✓	✓	S
Shepherding Algorithm	✓									✓	✓	S
Termite-Inspired Collective Construction Algorithm						✓	✓			✓	✓	H
Fish-inspired Goal Searching Algorithms		✓			✓					✓	✓	H
Gillespie Self-Assembly Algorithm		✓								✓	✓	H
Mergeable Modular Robots		✓								✓	✓	H
Density based Control												
Markov Chain-based Algorithms		✓	✓			✓				✓	✓	H
Smoothed Particle Hydrodynamics (SPH)		✓	✓							✓	✓	H
Optimal Transport based Algorithm		✓						✓		✓	✓	S
Distributed Optimization Algorithms												
Distributed Linear Programming		✓				✓				✓	✓	S
Distributed Convex Optimization		✓				✓			✓	✓	✓	S
Distributed Dynamic Programming						✓		✓				H
Sequential Convex Programming								✓		✓	✓	H
Distributed Auction						✓				✓	✓	H
Local Optimization Algorithms for Global Behavior												
Decentralized Model Predictive Control (DMPC)		✓						✓		✓		H
Formal Methods								✓		✓	✓	S
Sampling-based Motion-Planning Algorithms								✓				H
Centralized Optimization Algorithms												
MILPs and MINLPs		✓				✓		✓	✓		-	H
Linear and Convex Optimization						✓		✓	✓	✓	-	S
Markov Decision Processes (MDP)						✓		✓			-	H
Multi-Agent Traveling Salesman Problems			✓	✓	✓	✓					-	H
Multi-Armed Bandits				✓	✓	✓			✓	✓	-	S
Direct Methods for Optimal Control			✓	✓				✓			-	F
Multiagent Reinforcement Learning				✓		✓					-	H
Frontier Techniques				✓	✓						-	F
Network Flow Algorithms						✓		✓		✓	-	S
Combinatorial Motion Planning								✓		✓	-	S

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Geometric Algorithms												
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Game Theory based Algorithms						✓					-	S
Resource Allocation Systems								✓		✓	-	S
Bio-Inspired Algorithms												
Kilobot Self-Assembly Algorithm		✓								✓	✓	H
Optimotaxis Source-Searching Algorithm					✓					✓	✓	S
Beeclust Foraging Algorithm				✓						✓	✓	S
Shepherding Algorithm	✓									✓	✓	S
Termite-Inspired Collective Construction Algorithm						✓	✓			✓	✓	H
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Gillespie Self-Assembly Algorithm		✓								✓	✓	H
Mergeable Modular Robots		✓								✓	✓	H
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Smoothed Particle Hydrodynamics (SPH)		✓	✓							✓	✓	H
Optimal Transport based Algorithm		✓						✓		✓	✓	S
Distributed Optimization Algorithms												
Distributed Linear Programming		✓				✓				✓	✓	S
Distributed Convex Optimization		✓				✓			✓	✓	✓	S
Distributed Dynamic Programming						✓		✓				H
Sequential Convex Programming								✓		✓	✓	H
Distributed Auction						✓				✓	✓	H
Local Optimization Algorithms for Global Behavior												
Decentralized Model Predictive Control (DMPC)		✓						✓		✓		H
Formal Methods								✓		✓	✓	S
Sampling-based Motion-Planning Algorithms								✓				H
Centralized Optimization Algorithms												
MILPs and MINLPs		✓				✓		✓	✓		-	H
Linear and Convex Optimization						✓		✓	✓	✓	-	S
Markov Decision Processes (MDP)						✓		✓			-	H
Multi-Agent Traveling Salesman Problems			✓	✓	✓	✓					-	H
Multi-Armed Bandits				✓	✓	✓			✓	✓	-	S
Direct Methods for Optimal Control			✓	✓				✓			-	F
Multiagent Reinforcement Learning				✓		✓					-	H
Frontier Techniques				✓	✓						-	F
Network Flow Algorithms						✓		✓		✓	-	S
Combinatorial Motion Planning								✓		✓	-	S

Conclusion

- Majority of algorithms tailored to either low-level tasks or high-level applications.
- Few algorithms (e.g. APF) can be used for the entire range.
- Very few algorithms are mature and field-tested.

Algorithms	Tasks: Aggregation	Pattern Formation	Coverage	Area Exploration	Goal Searching	Task Allocation	Collective Transport	Motion Planning	Distributed Estimation	Features: High Scalability	Low Bandwidth Use	Maturity
Consensus	✓	✓	✓						✓	✓	✓	H
Artificial Potential Functions (APF)	✓	✓	✓	✓		✓	✓	✓		✓	✓	F
Distributed Feedback Control	✓	✓							✓	✓	✓	F
Geometric Algorithms												
Voronoi-based Algorithms	✓		✓	✓				✓		✓	✓	H
Circumcenter Algorithms	✓	✓								✓	✓	S
Bearing-only Algorithms	✓	✓								✓	✓	H
Maze Searching Algorithms								✓		✓	✓	S
Leader-Follower (LF) Algorithms		✓								✓	✓	S
Velocity Obstacle (VO) based Algorithms								✓		✓	✓	F
State Machines and Behavior Composition												
Automata-based Algorithms						✓			✓	✓	✓	S
Behavior Composition						✓	✓					H
Petri Networks						✓					-	H
Game Theory based Algorithms						✓					-	S
Resource Allocation Systems								✓		✓	-	S
Bio-Inspired Algorithms												
Kilobot Self-Assembly Algorithm		✓								✓	✓	H
Optimotaxis Source-Searching Algorithm					✓					✓	✓	S
Beeclust Foraging Algorithm				✓						✓	✓	S
Shepherding Algorithm	✓									✓	✓	S
Termite-Inspired Collective Construction Algorithm						✓	✓			✓	✓	H
Fish-inspired Goal Searching Algorithms		✓			✓					✓	✓	H
Gillespie Self-Assembly Algorithm		✓								✓	✓	H
Mergeable Modular Robots		✓								✓	✓	H
Density based Control												
Markov Chain-based Algorithms		✓	✓			✓				✓	✓	H
Smoothed Particle Hydrodynamics (SPH)		✓	✓							✓	✓	H
Optimal Transport based Algorithm		✓						✓		✓	✓	S
Distributed Optimization Algorithms												
Distributed Linear Programming		✓				✓				✓	✓	S
Distributed Convex Optimization		✓				✓			✓	✓	✓	S
Distributed Dynamic Programming						✓		✓				H
Sequential Convex Programming								✓		✓	✓	H
Distributed Auction						✓				✓	✓	H
Local Optimization Algorithms for Global Behavior												
Decentralized Model Predictive Control (DMPC)		✓						✓		✓		H
Formal Methods								✓		✓	✓	S
Sampling-based Motion-Planning Algorithms								✓				H
Centralized Optimization Algorithms												
MILPs and MINLPs		✓				✓		✓	✓		-	H
Linear and Convex Optimization						✓		✓	✓	✓	-	S
Markov Decision Processes (MDP)						✓		✓			-	H
Multi-Agent Traveling Salesman Problems			✓	✓	✓	✓					-	H
Multi-Armed Bandits				✓	✓	✓			✓	✓	-	S
Direct Methods for Optimal Control			✓	✓				✓			-	F
Multiagent Reinforcement Learning				✓		✓					-	H
Frontier Techniques				✓	✓						-	F
Network Flow Algorithms						✓		✓		✓	-	S
Combinatorial Motion Planning								✓		✓	-	S

F. Rossi, S. Bandyopadhyay, M. Wolf, M. Pavone, "Review of Multi-Agent Algorithms for Collective Behavior: a Structural Taxonomy", https://arxiv.org/abs/1803.05464

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